Comment on "Drag Coefficient of Spheres in Continuum and Rarefied Flows"

Michael J. Walsh*
NASA Langley Research Center, Hampton, Va.

HENDERSON[1] has published a new drag coefficient correlation which reportedly gives better predictions of experimental sphere drag data[2] than previous methods. The approach uses two equations for the drag coefficient (C_D): one for relative Mach number (M_r) less than one, and one for M_r greater than 1.75. For the $M_r = 1.00\text{-}1.75$ range, the method uses a linear interpolation between C_D evaluated at $M_r = 1.00$ and 1.75. The purpose of the present note is to compare Henderson's C_D correlations[1] with the method developed by the present author.[3,4] The method developed in Ref. 3 emphasized increased accuracy of C_D predictions for $M_r < 2.0$ and relative Reynolds number $Re_r < 200$. This relative Mach number and Reynolds number range is of particular importance in laser velocimetry applications.

The C_D methods of Refs. 1 and 3 are empirical, based on experimental sphere C_D data. Therefore, relative evaluation must be based on their prediction accuracy for experimental sphere C_D data. As noted in Refs. 1 and 3, the most extensive sphere C_D data are those of Bailey and Hiatt,[2] which include data for $M_r = 0.12\text{-}6.0$ at $Re_r > 200$, and $M_r = 1.0\text{-}6.0$ at $Re_r = 20\text{-}200$. In addition to the above data, Zarin[5] has made subsonic sphere C_D data measurements for $Re_r < 100$.

Figures 1 and 2 show the prediction accuracy of the Henderson[1] and the Walsh[3] C_D correlations, respectively, using the data of Bailey and Hiatt[2] and Zarin.[5] The figures indicate that the two methods give equivalent prediction accuracy at $M_r = 2.0$, but that the method of Ref. 3 gives better "predictions" of the C_D data at lower relative Mach number and Reynolds number.

The following comparison of the two C_D methods addresses reasons for equivalent prediction accuracy in certain M_r and Re_r ranges, and problems with the Henderson C_D method[1] that could lead to decreased accuracy in other M_r and Re_r ranges.

For $M_r > 1.75$, both approaches use an equation of the form

$$C_D = (C_{DC} + KC_{DFM})/(1+K) \tag{1}$$

where C_{DC} is the continuum value and C_{DFM} is the free molecular value of the drag coefficient. The two C_D methods differ only in the equation used for K, where K is the parameter that fits Eq. (1) to experimental data. Since both methods use the same experimental data and Eq. (1), it is not surprising that they give equivalent prediction accuracy for $M_r > 1.75$.

For $M_r = 1.0\text{-}1.75$, Henderson[1] uses a linear interpolation between C_D evaluated at $M_r = 1.0$ and $M_r = 1.75$. The data of Bailey and Hiatt[2] indicate that C_D is not a linear function of M_r in the range $M_r = 1.0\text{-}1.75$. This explains the decreased accuracy of the Henderson C_D method[1] at $M_r = 1.25$ as shown in Fig. 1. It is suggested that the same procedure that Henderson[1] used for $M_r > 1.75$ should have been used for $M_r = 1.0$ to 1.75. This is the procedure used in Ref. 3.

Received Feb. 7, 1977.
Index categories: Multiphase Flow; Nozzle and Channel Flow.
*Aerospace Engineer, Applied Fluid Mechanics Section, High-Speed Aerodynamics Division. Member AIAA.

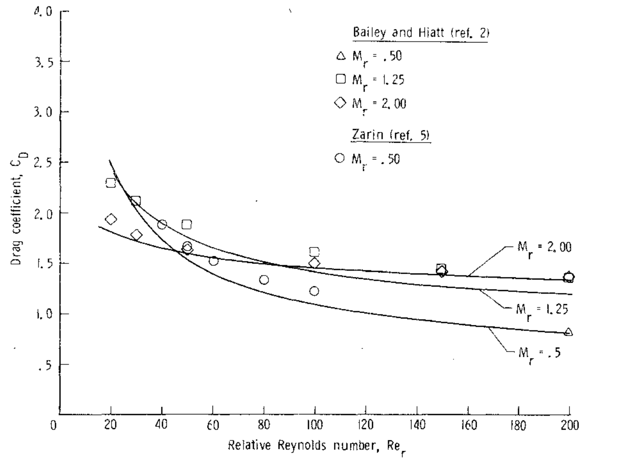

Fig. 1 Comparison of Henderson C_D method[1] with experimental sphere C_D data.

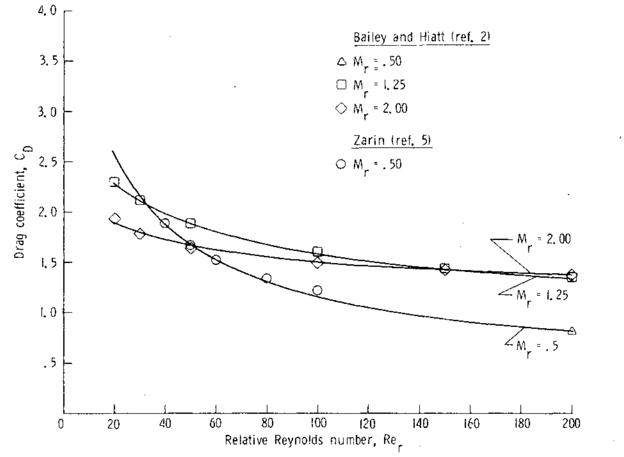

Fig. 2 Comparison of Walsh C_D method[3,4] with experimental sphere C_D data.

For $M_r < 1$ Henderson[1] employs an equation quite different from Eq. (1); whereas, the present author uses Eq. (1) for $M_r \geq .1$. Figures 1 and 2 show that both C_D methods give equivalent prediction accuracy of the Bailey and Hiatt[2] data at $Re_r = 200$ and $M_r = 0.5$; however, the Henderson C_D method[1] exhibits a decrease in accuracy at the lower relative Reynolds number at $M_r = 0.5$. Both methods approach the incompressible continuum C_D values as M_r is decreased; however, while comparing the two C_D methods it was found that the present author's method presented in Ref. 3 exhibits a discontinuity at relative Reynolds numbers less than 50 and $M_r = 0.1$. The value of $M_r = 0.1$ is the dividing point between using Eq. (1), which accounts for compressibility and rarefaction effects, and the incompressible continuum C_D equation given below

$$C_D = 24/Re_r(1 + 0.15\ Re_r^{0.687}) \tag{2}$$

The discontinuity for $Re_r < 50$ and $M_r = 0.1$ was due to the flow being in the slip regime rather than the continuum regime. According to Emmons,[6] the Knudson number (K) defined as

$$K = M_r/\sqrt{Re_r} \quad \text{if } Re_r > 1 \tag{3a}$$

$$K = M_r/\ Re_r \quad \text{if } Re_r < 1 \tag{3b}$$

must be less than 0.01 for continuum flow. By using Eq. (3) a maximum value of M_r for continuum incompressible flow can be computed as follows

$$M_{r\max} = 0.01\ \sqrt{Re_r} \quad \text{if } Re_r > 1$$

$$M_{r\max} = 0.01\ \ Re_r \quad \text{if } Re_r < 1 \tag{4}$$

The C_D method presented in Ref. 3 should be modified as follows to give a smooth transition between Eqs. (1) and (2)

for $M_r \geq 0.1 \quad C_D = C_{DI} = C_{DC} + (C_{DFM} - C_{DC})\, e^{-ARe_r^N}$

for $M_r < M_{r\max} < 0.1 \quad C_D = C_{D2} = 24/Re_r(1 + 0.15\ Re_r^{\ 0.687})$

for $M_{r\max} < M_r < 0.1 \quad C_D = \dfrac{C_{DI} - C_{D2}}{0.1 - M_{r\max}}\,(M_r - M_{r\max}) + C_{D2}$ (5)

In summary, the sphere drag correlation presented by Henderson[1] and the present author[3] should provide similar prediction accuracy of C_D for $M_r > 1.75$; however, the present author's C_D method[3] gives better predictions of C_D for $M_r < 1.75$. However, the method originally presented in Ref. 3 should be modified for $M_r < 0.1$ as described herein.

References

[1]Henderson, C. B., "Drag Coefficients of Spheres in Continuum and Rarefied Flows," *AIAA Journal*, Vol. 14, June 1966, pp. 707-708.

[2]Bailey, A. B. and Hiatt, J., "Free-Flight Measurements of Sphere Drag at Subsonic, Transonic, Supersonic, and Hypersonic Speeds for Continuum, Transition, and Near-Free-Molecular Flow Conditions," Arnold Engineering Development Center, Tullahoma, Tenn., AEDC-TR-70-291, March 1971.

[3]Walsh, M. J., "Drag Coefficient Equations for Small Particles in High Speed Flows," *AIAA Journal*, Vol. 13, Nov. 1975, pp. 1526-1528.

[4]Walsh, M. J., "Influence of Particle Drag Coefficient on Particle Motion in High-Speed Flow with Typical Laser Velocimetry Applications," NASA TN D-8120, Feb. 1976.

[5]Zarin, N. A., "Measurement of Non-Continuum and Turbulence Effects on Subsonic Sphere Drag," NASA CR-1585, June 1970.

[6]Emmons, H. W., Ed., *Fundamentals of Gas Dynamics, Vol. III. High Speed Aerodynamics and Jet Propulsion*, Princeton University Press, Princeton, N.J., 1958, p. 689.

Reply by Author to M. J. Walsh

C. B. Henderson*

Atlantic Research Corporation, Alexandria, Va.,

THE stated purpose of Walsh's comment is to compare his correlation[1,2] of drag coefficients with one proposed by this author.[3] This comparison is accomplished within a restricted range of Mach numbers and Reynolds numbers. Figures 1 and 2 of Walsh's comment show that, at Reynolds numbers Re between 20 and 200 and at Mach numbers M of 0.5 and 1.25, his correlation is more accurate. This author is in agreement with this conclusion; a quantitative comparison of the two correlations with the experimental data of Bailey and Hiatt[4] shows that the maximum error within this range of conditions is reduced from 16% to 7% by use of the Walsh correlation. It should be pointed out, however, that this author's correlation was intended to be used over a much wider range of Reynolds numbers and Mach numbers, namely $0 < M < 6$ and $0 < Re < Re_{cr}$, where Re_{cr} is the Reynolds number at which turbulence produces a sudden reduction in the drag coefficient. The range of Reynolds numbers and Mach numbers covered by the two respective correlations is shown in Fig. 1.

Received March 17, 1977.

Index categories: Multiphase Flow; Nozzle and Channel Flow.

*Vice President. Associate Fellow AIAA.

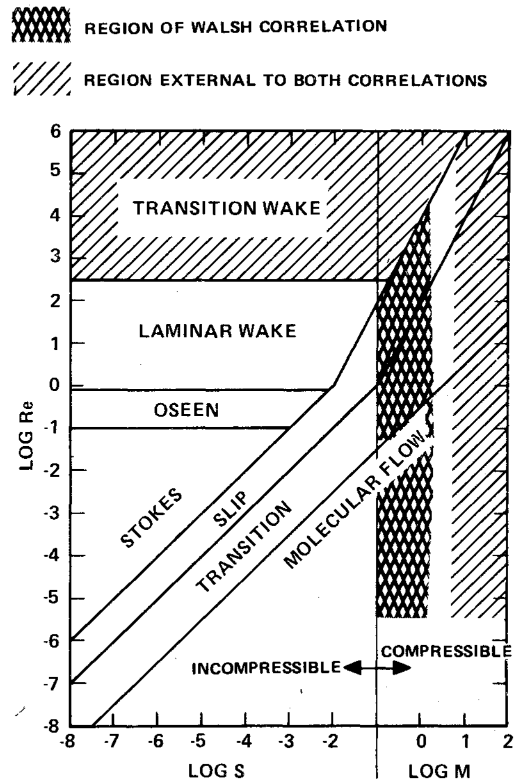

Fig. 1 Regions of applicability of C_D correlations.

It should be of interest to see how the two types of correlations compare in regions outside of the very narrow one in which Walsh made his comparison. Since Walsh[1,2] only gives correlating parameters for $0.1 < M < 2$, a quantitative comparison can be made only within that range. At higher Reynolds numbers, $200 < Re < 10^4$, the two correlations give the same maximum percentage deviation from the data of Bailey and Hiatt; specifically 7-8% at $0.1 < M < 0.5$ and 15% at $0.5 < M < 2.0$.

At Reynolds numbers below 20, there are no experimental data with which to compare; instead the limit of C_D in the molecular flow regime approached by both correlations at low Re can be investigated. Both Walsh and this author employ a theoretical equation to calculate C_D in the molecular flow regime. The equations used differ for two reasons. First, Walsh employs an equation for diffuse reflection only, while this author considers both diffuse and specular reflection; neglecting specular reflection results in C_D values which are higher by 3%. Second, Walsh's expression for diffuse reflection contains an error. The first term of Walsh's diffuse reflection equation (Eq. (9a), p. 4 of Ref. 2) is

$$(1+2s^2)\exp\left(\frac{-s^2/2}{\sqrt{\pi s^3}}\right)$$

This apparently contains a typographical error, since the numerical values given by Walsh are in agreement with the equation given in Walsh's basic reference (Schaaf and Chambre[5]), the first term of which is

$$\frac{(1+2s^2)}{\sqrt{\pi s^3}}\exp\,(-s^2/2)$$

Schaaf and Chambre[5] have a typographical error in their first term, however, as shown by comparison to the equation given by Stalder and Zurick,[6] the original authors of the equation.